The improvement of a variational level set formulation for image segmentation

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Abstract

This paper shows a new improvement variational formulation for geometric active contours that make sure the level set function to be close to a signal distance function. The variational formulation consists of an internal energy term that penalizes the deviation of the level set function from a signal distance function. An external energy term that drives the motion of the zero level set toward the object boundaries. Therefore eliminates the need of the costly re-initialization procedure. Upon simulation experiments present that the method is fast and applicable way for application in image segmentation. This method not only simplifies the calculation, but also the iteration can be set longer than the traditional method in time-step so that the evolution of the curve faster. Its flexibility in initializing level set function makes the selection of initial contour has more freedom, and calculations are a lot simpler.

Keywords: active contours, level set, image segmentation

1 Introduction

Image segmentation methods, for instance, a combination of geometric active contour models and level set, and so on, have been attracting a lot of interest. Geometric active contour models, presented separately by the Casellos and Malladi [1, 2], use the geometric characteristics of contour curves to establish energy functions of the movement of the contour curve .Using different contour curve energy functions to guide different contour curve motions will result in different image segmentations. Level Set method is an important numerical method for partial differential image analysis [3-5]. It presents low dimensional contour curves with the aid of a high-dimensional surface. It is an alternative way to present that a contour curve in the form of a function of the surface with high-dimensional zero level set (a set of all points of where the level set function has a value of zero). In Level Set Method, an equation of a contour curves movement is transferred into a higher dimensional level set function of partial differential equations [6-9]. The most significant advantage of presenting contour curves in this way is that even the contour curve concealed inside the level set function has changed (combined or split), the level set function remains a valid function. Therefore, the Geometric Active Contour models based on curve theory and level set method can naturally deal with the changes of contour curve topology, and also can be implemented numerically in a fixed grid. How to design an effective energy function of contour curves to obtain correct or desired results of image segmentations is one of the most actively investigated topics in image segmentation methods based on geometric active contour models and level set combining.

2 Signal distance function and the traditional level set method

Given a closed contour curves on the plane, if $d(x,y)$ is the shortest distance from point (x,y) to the contour curve, it is then denoted as Signal Distance Function, or SDF in abbreviation. Here we specify that $d(x,y)<0$ for points inside the contour curve, and that $d(x,y)>0$ for points outside contour curve. Let s be the arc length parameter of the contour curve C , and continuous function $\Phi(t,x,y):R^2\times R\to R$ is the implicit expression of the closed contour curve $C(s,t):0\le s\le 1$ at time t . Which means, at time t, the Zero level set of $C(s,t)$ corresponding to $\Phi(t,x,y)$ is $C(t,s)=\{(x,y)|\Phi(t,x,y)=0\}$. A partial differential equations expressed in level set contour curve representation:

$$\frac{\partial\Phi}{\partial t}+F|\nabla\Phi|=0\,, \tag{1}$$

is called level set equation, where F is function of speed in equation [4]. In the image segmentation, F is related to the image itself and the level set function Φ. In traditional level set method, the level set function may be unstable in the process of evolution, resulting in contours farther away from the target. In order to avoid such a situation, the common practice is to initialize function Φ to SDF before

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evolution. In the process of evolution, in order to avoid the "drift" of the contour curve position, the level set function should be maintained as a signal distance function. Thus the level set function needs to be corrected continuously, that is to re-initialize so that it remains signal distance function. This is usually obtained through the steady state solution of heat conduction equation as following:

$$\frac{\partial\Phi}{\partial t} = sign(\Phi_0)(1-|\nabla\Phi|). \tag{2}$$

The Φ_0 here is the level set function to be re-initialize. A very time-consuming step in level set method is to re-initialize the level set function to a signal distance function. Shortening the time and improving the accuracy of distance values will help to improve the efficiency curve evolution. Therefore many people come up with many methods to re-initialize the level set [10-13], and most of the methods are based on partial differential equations. However, there is no simple guideline for when and how to re-initialize the level set function to a signal distance function [14-16]. Based on this, it propose an approach with no need to re-initialize the level set function to SDF.

3 Equation with no need to re-initialize the level set function

As previously described, in the evolution of the curve, keeping the level set function in evolution close to a signal distance function is critical, especially in the neighborhood of the zero level set. As is well-known, signal distance function Φ must satisfy Partial Differential Equations $|\nabla\Phi|=1$. On the contrary, all Φ which satisfy $|\nabla\Phi|=1$, are signal distance function plus a constant. Therefore, it will define the following integral:

$$\mathrm{P}(\Phi) = \int_\Omega \frac{1}{2}\left(|\nabla\Phi|-1\right)^2 dxdy, \tag{3}$$

to measure the closeness of function Φ and the signed distance function in $\Omega \in R\times R$. Plugging $\mathrm{P}(\Phi)$ into energy function, we will get:

$$\varepsilon(\Phi) = \mu\mathrm{P}(\Phi) + \varepsilon_m(\Phi), \tag{4}$$

where $\mu > 0$ is the weight parameter, and is used to adjust the weight of $\mathrm{P}(\Phi)$. $\mu\mathrm{P}(\Phi)$ can be regarded as internal energy function, and it ensures function Φ is not away from signed distance function. $\varepsilon_m(\Phi)$ is some energy to drive the zero level set curve of Φ to move towards the boundaries, so it can be treated as external energy. Energy function ε_m is defined as a function related to image data. Define the external energy function for function $\Phi(x,y)$ as follows:

$$\varepsilon_{g,\lambda,v}(\Phi) = \lambda L_g(\Phi) + vA_g(\Phi), \tag{5}$$

where, $\lambda > 0, \lambda, v$ are both constants, they can be treated as weight parameters; the length and size $L_g(\Phi), A_g(\Phi)$ are expressed as Equations (6) and (7) as follows:

$$L_g(\Phi) = \int_\Omega g\delta(\Phi)|\nabla\Phi| dxdy. \tag{6}$$

To better understand the geometric meaning of $L_g(\Phi)$, let the zero level set of Φ to be represented by Derivative function $C(p), p\in[0,1]$. As we know, the length of $C(p)$ is calculated using equation, $ds = g(C(p))|C'(p)|dp$, and the length of zero level set curves of Φ can be calculated through $L_g(\Phi)$. In Equation (6), δ is a Dirac Function.

$$A_g(\Phi) = \int_\Omega gH(-\Phi)dxdy, \tag{7}$$

where H is Heaviside Function. $A_g(\Phi)$ functions as the acceleration of the evolution of the curve., In particular, when g equals 1, the energy function of the equation represents area $\Omega_\Phi^- = \{(x,y)|\Phi(x,y)<0\}$ [10], thus, energy function $A_g(\Phi)$ can be treated as the weighted area of the area Ω_Φ^-. And as the parameter of $A_g(\Phi)$, v can be either positive or negative, and Its value depends on the relative position of the target area and the initial outline of interest. If the initial outline is outside the target, v is positive, and it will give the outline of shrinking faster if the initial contour inside the target; v is negative, it will accelerate the expansion of the contour.

The function g in Equation (6) and (7) is edge detection function. It is related to the borders of target image and it is used to control the velocity contour curve. Let I represents an image, edge detection function g is then expressed as:

$$g = \frac{1}{1+|\nabla G_\sigma * I|^2}, \tag{8}$$

where, G_σ is Gaussian kernel with standard deviation σ. When the active contour is located on the edge of the image, the gradient of the Gaussian mode of the image at this time is large, and therefore, g is small. Evolution velocity of the curve is almost zero, and the curve is stopped at the edge of the image.

In summary, we can define the complete energy function as follows:

$$\varepsilon(\Phi) = \mu\mathrm{P}(\Phi) + \varepsilon_{g,\lambda,v}(\Phi), \tag{9}$$

where, external energy $\varepsilon_{g,\lambda,v}$ drives zero level set towards the target contour; internal energy $\mu\mathrm{P}(\Phi)$ ensures that curve does not deviate from the signed distance function during evolution. By minimizing the energy function, we can make the contour curve move to the image object boundary, and the energy function and velocity function of contour curves are expressed through steepest gradient descent method as follows:

$$\frac{\partial\Phi}{\partial t} = -\frac{\partial\varepsilon}{\partial\Phi}. \tag{10}$$

Using calculus of variations, we get equation:

$$\frac{\partial\varepsilon}{\partial\Phi} = -\mu\left[\Delta\Phi - div\left(\frac{\nabla\Phi}{|\nabla\Phi|}\right)\right] - \lambda\delta(\Phi)divg\left(\frac{\nabla\Phi}{|\nabla\Phi|}\right) - vg\delta(\Phi),$$

where, Δ is Laplacian. In order to minimize the function, it needs to satisfy the Euler - Lagrange equations $\frac{\partial\varepsilon}{\partial\Phi} = 0.$ Through the steepest descent method Equation (10) to get the level set function evolution equations:

$$\frac{\partial\Phi}{\partial t} = \mu\left[\Delta\Phi - div\left(\frac{\nabla\Phi}{|\nabla\Phi|}\right)\right] + \lambda\delta(\Phi)div\left(g\frac{\nabla\Phi}{|\nabla\Phi|}\right) + vg\delta(\Phi), \tag{11}$$

where the second and the third part of right side come from external energy, to drive zero level curves to evolve towards target boundaries. As for the first part, relating it to internal energy $\mu\mathrm{P}(\Phi)$, we get:

$$\Delta\Phi - div\left(\frac{\nabla\Phi}{|\nabla\Phi|}\right) = div\left[\left(1 - \frac{1}{|\nabla\Phi|}\right)\nabla\Phi\right]. \tag{12}$$

where $\left(1 - \frac{1}{|\nabla\Phi|}\right)$ is the Thermal Diffusivity. If $|\nabla\Phi| > 1$, diffusivity is positive, which is the diffusivity in general meaning, i.e., it will make Φ more flat, and gradient $|\nabla\Phi|$ smaller. If $|\nabla\Phi| < 1$, then the diffusivity is increased towards the opposite direction because of this gradient. Through diffusivity adjustments like this, $|\nabla\Phi| \approx 1$, so that level set function Φ is always close to signed distance function, therefore there is no need to re-initialize the level set function.

4 Numerical implementation of the curve evolution equation

4.1 SOLVING PARTIAL DIFFERENTIAL EQUATIONS

Because there is no need to re-initialize the level set function, we no longer using the traditional level set methods for solving inverse term difference. Instead, we use finite difference methods to solve the Equation (11). In practice, regularized Dirac function $\delta(x)$ is used. Dirac function $\delta(x)$ is defined as:

$$\delta_\varepsilon(x) = \begin{cases} 0 \\ \frac{1}{2\varepsilon}\left[1 + \cos\left(\frac{\pi x}{\varepsilon}\right)\right], |x| \le \varepsilon \end{cases}, \tag{13}$$

where ε is the width defined by $\delta(x)$, which affects the detection capability of the contour line, in experiments it is generally between 1.2 and 1.5.

In simulation, let $\varepsilon = 1.5$, all the spatial partial derivative $\frac{\partial\Phi}{\partial x}$ $\frac{\partial\Phi}{\partial y}$ related with locations are replaced with central difference:

$$\Phi_x^k = \frac{\Phi_{i+1,j}^k - \Phi_{i-1,j}^k}{2\tau}, \Phi_y^k = \frac{\Phi_{i,j+1}^k - \Phi_{i,j-1}^k}{2\tau}.$$

Time partial derivatives $\frac{\partial\Phi}{\partial t}$ is replaced with forward difference:

$$\frac{\partial\Phi}{\partial t} = \frac{\Phi^{k+1} - \Phi^k}{\tau}.$$

Equation (11) can be simply expressed as:

$$\frac{\Phi_{i,j}^{k+1} - \Phi_{i,j}^k}{\tau} = E(\Phi_{i,j}^k), \tag{14}$$

where $E(\Phi_{i,j}^k)$ is the result of the replacement of the partial derivative using differences of Equation (11). Therefore, we get iterative curve evolution equation, which can be expressed as the following form:

$$\Phi_{i,j}^{k+1} = \Phi_{i,j}^k + \tau E(\Phi_{i,j}^k). \tag{15}$$

4.2 DETERMINE THE INITIAL LEVEL SET FUNCTION

In a general level set in the traditional method, the level set function Φ is defined as signal distance function. Curve evolution process requires re-initializing Φ to maintain as signal distance function. In this model, because of the adjustment of internal energy $\mu P(\Phi)$, not only curve evolution process does not require to re-initialize the level set function, but also the initial level set function Φ_0 does not need to be signed distance function, and it can be obtained in various ways. Here it is defined as follow: let

R_0 be a subset of the image range R, and ∂R_0 is the set of all the points of contours of R_0. It can be identified by simple morphological operations. The initial level set function Φ_0 can be defined as Equation (16):

$$\Phi_0(x,y)=\begin{cases}-p,(x,y)\in R_0-\partial R_0\\ 0,(x,y)\in \partial R_0\\ p,(x,y)\in R-R_0\end{cases}, \qquad (16)$$

where, p is a positive constant. In general, $p>2\varepsilon$, ε is the regularized Dirac function parameter, in practice, $5<p<6$.

Different from traditional calculation of signal distance function using initial contour, the initial level set function can start from any subinterval R_0. This initialization region-based approach is not only easy to calculate, but also is more flexible in practical applications. Although this approach allows the level set function to initialize initially deviates significantly from the signed distance function, but due to the internal energy $\mu P(\Phi)$ in the energy function, Φ can be approximately equal to the signal distance function in the vicinity of its zero level set, so there is not much affection on boundary location.

4.3 ITERATION STEP

It is well-known that a larger time step can speed up the convergence rate of iterations which can accelerate the evolution of the curve, but too large of the time step often cause errors in determining the position of the boundary. In the traditional level set method, in order to guarantee the stability of the iterative process, the time step τ is generally very small, for instance 0.1 or .0.05. In the method of this paper, the time step can be selected larger than traditional methods, which can greatly improve the convergence speed. Most experiments usually take compromise on the choice of the step size and contour positioning accuracy. In our experiment we selected $\tau\le 10$, and when $\tau\mu\le 1/4$, level set curve can evolve relatively stable.

4.4 ALGORITHM STEPS

Step 1: set the initial contour, it can be arranged in a circle, rectangle, or a non-closed curve, using Equation (16) to calculate the initial value of the level set function.

Step 2: using an iterative Equation (15), calculate $\Phi_{i,j}^{k+1}$.

Step 3: updated contour curves, obtain image segmentation.

5 Image segmentation experiments

Matlab is used here as a split tool. Using the method described in the paper, image segmentation for multiple simulation experiments are obtained with good:

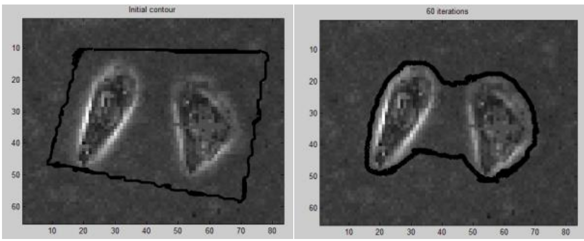

FIGURE 1a: original image, 1b: iterations 60 times

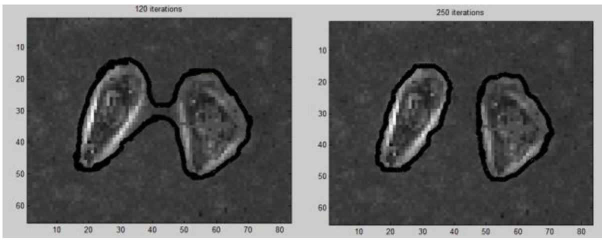

FIGURE 1c: iteration 120 times, 1d: 250 times

Figure 1 is an original image size of 64×80 pixels in size cell microscopic images, the boundary is blurry, and the yellow border is the initial contour. The parameters are $\tau=5$, $\lambda=5$, $\mu=0.04$, $\nu=1.5$. Figures 1b, 1c and 1d are the level set curve evolution results, and iterations were 60 times, 120 times and 250 times, respectively. As it can be seen, although the initial evolution curve is relatively far from the contour lines, after 250 iterations, the level set curve is a good fit for the image contours.

6 Conclusions

After setting the initial level set based on a regional basis, we have introduced of an internal energy to ensure that in the direction of the curve evolution. There is no need to re-initialize the level set function to a signal distance function. Thus the work overcomes inconvenience of the traditional level set method, which repeatedly re-initializes the level set function [17]. This method not only simplifies the calculation, and also the iteration can be set longer than the traditional method in time-step so that the evolution of the curve faster. Its flexibility in initializing level set function makes the selection of initial contour has more freedom, and calculations are a lot simpler. Experiments show that the proposed method can get satisfactory results in the images with fuzzy boundaries and concave. However, this method has shortcomings, such as the setting of initial contour cannot cross homogeneous region. Additionally, in some deep concaves, it will often fail to detect. Further improvement is needed.

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